

# Energy dependence of the single spin asymmetries in inclusive pion production processes

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## Abstract

Recent data from E925 Collaboration at Brookhaven National Laboratory show a significant energy dependence of the single-spin left-right asymmetry in inclusive hadron production in hadron-hadron collisions. We analyzed the experimental results and show that the observed energy dependence can be reproduced naturally in the picture proposed in a previous Letter. We fixed all the parameters at the Fermilab E704 energy and calculate the asymmetry at the BNL E925 energy. We compare the results with the data and make predictions for experiments at even higher energies.

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The existence of striking single-spin left-right asymmetries ( $A_N$ ) in inclusive hadron production in hadron-hadron collisions with transversely polarized beam has attracted much attention in the past years. In contrast to the leading order theoretical prediction made[1] in 1978, the data[2] show that the produced hadrons exhibit left-right asymmetries up to 40% in the fragmentation region. A number of theoretical approaches[4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16] have been made, the aim of them is to explain why such striking effects exist. But the mechanisms which lead to such asymmetries are still in debate. New measurements at high energies are underway. It is interesting to note the recent data from the E925 Collaboration at Brookhaven National Laboratory (BNL) for pion production in  $pp$  collisions[17] at  $p_{inc} = 22$  GeV/c. Compared with the results obtained earlier by Fermilab E704 Collaboration[3] at  $p_{inc} = 200$  GeV/c, the data from BNL E925 show the following characteristics: First, the E925 data confirms the striking features of the E704 data: (a) the magnitudes of  $A_N$  are approximately zero for small  $x_F$ , they increase monotonically with increasing  $x_F$  and reach about 40% at  $x_F \approx 0.8$ ; (Here,  $x_F \equiv 2p_{\parallel}/\sqrt{s}$ ,  $p_{\parallel}$  is the component of the momentum of the produced meson in the center of mass frame of the colliding hadron system parallel to the direction of motion of the incident hadron;  $s$  is the center of mass energy squared.) (b) the sign of  $A_N$  for  $\pi^+$  in  $pp$  collisions is positive and that for  $\pi^-$  is negative. Second, at the same time, the E925 data show also the following striking energy dependence: The  $|A_N|$ 's at the E925 energy start to rise much later than those at the E704 energy. They start to rise at about  $x_F \approx 0.5$  while those at the E704 energy start at  $x_F \approx 0.3$ . It should be helpful in distinguishing different mechanisms to see whether such an energy dependence can be reproduced.

One of the theoretical approaches to explain the asymmetries in the last years has been proposed by the Berliner group[6, 7, 8, 16]. The major characteristics of the picture are: (1) The produced mesons are divided into two categories: the “direct-formation” or “direct-fusion” part and the rest, where the former denotes those mesons which are directly produced and contain the valence quarks from the incident hadron. It has been shown that the former can be described by the direct-fusion process  $q_v + \bar{q}_s \rightarrow M$  and contributes to the single-spin left-right asymmetry but the latter does not. (2) The left-right asymmetry for a meson from  $q_v(\uparrow) + \bar{q}_s \rightarrow M$  arises because of the orbital angular momentum of the valence quark in the polarized proton and the “surface effect” during the hadron-hadron collisions. The picture has been applied to different processes at E704 energy. A good agreement with the

various available data has been obtained with only two adjustable parameters[16]. It should be interesting to see whether this picture can give rise to the energy dependence observed by E925 Collaboration compared to the E704 data.

In this note we study this problem and show that the energy dependence can easily be reproduced by the picture in [7] without adjusting or introducing any further parameters. We used the values of the two parameters fixed at the E704 energy to do the calculations at the E925 energy. We compare the results with the data and extend the discussion to even higher energy region.

We now start the discussion with a brief summary of the formulation of the picture in [7]. We use the same notations as in [7] and denote the number densities of the mesons from the direct-fusion process  $q_v + \bar{q}_s \rightarrow M$  and the rest by  $D(x_F, s|M)$  and  $N_0(x_F, s)$  respectively. Empirical facts (see the references given in [7, 8, 16]) show that  $D(x_F, s|M)$  can be obtained from the quark distributions  $q_v(x_v)$  and  $\bar{q}_s(x_s)$  of the corresponding flavors. (Here,  $x_v$  and  $x_s$  are the momentum fractions carried by  $q_v$  and  $\bar{q}_s$  respectively.) Since energy-momentum conservation in the process requires that  $x_v \approx x_F$  and  $x_s \approx x_0/x_F$  (where  $x_0 = m^2/s$ ,  $m$  is the mass of the produced meson), it has been obtained that,

$$D(x_F, s|M) = \kappa_M q_v(x_F) \bar{q}_s(x_0/x_F), \quad (1)$$

where  $\kappa_M$  is a constant which is fixed by one data point on the cross-section in the large  $x_F$  region of the unpolarized reaction[18]. More precisely, for  $\pi^+$  and  $\pi^-$ , we have,

$$D(x_F, \pi^+|s) = \kappa_\pi u_v(x_F) \bar{d}_s(x_0/x_F), \quad (2)$$

$$D(x_F, \pi^-|s) = \kappa_\pi d_v(x_F) \bar{u}_s(x_0/x_F). \quad (3)$$

It should be emphasized that  $D(x_F, M|s)$  dominates the large  $x_F$  region, while the non-direct-formation part  $N_0(x_F, M|s)$  comes mainly from the interaction of the seas (sea quarks, sea anti-quarks and gluons) of the two colliding hadrons and is dominant in small  $x_F$  region.

In reactions with transversely polarized beams, the single-spin left-right asymmetry  $A_N$  comes from the direct-formation part, and is given by[7, 16],

$$A_N(x_F, M|s) = \frac{C \Delta D(x_F, M|s)}{N_0(x_F, M|s) + D(x_F, M|s)}, \quad (4)$$

where  $\Delta D(x_F, M|s) = \kappa_M \Delta q_v(x_F|tr) \bar{q}_s(x_0/x_F)$  is the difference between the number density of  $M$  from  $q_v + \bar{q}_s \rightarrow M$  where the  $q_v$  is polarized in the same and that in the opposite direction as the proton in reaction with upward polarized proton beam;  $\Delta q_v(x_F|tr) = q_v^+(x_F, \uparrow) - q_v^-(x_F, \uparrow)$  and  $q_v^\pm(x_F, \uparrow)$  is the number density of  $q_v$  in a transversely polarized proton where the polarization of  $q_v$  is the same (+) as or opposite (-) to the proton. We note that the  $\Delta q_v(x|tr)$  introduced in [7] is nothing else but the transversity distribution  $\delta q_v(x)$  now discussed frequently in the literature. The constant  $C$  is the difference between the probability for  $M$  produced in  $q_v + \bar{q}_s \rightarrow M$  to go left and that to go right if the  $q_v$  is upward polarized. It is the second parameter in the model and is fixed[7] to be  $C = 0.6$  by one  $A_N$  data in the large  $x_F$  region.

As has been emphasized above, for large  $x_F$ ,  $D(x_F, M|s) \gg N_0(x_F, M|s)$ . Hence,

$$A_N(x_F, M|s) \rightarrow \frac{C \Delta D(x_F, M|s)}{D(x_F, M|s)} = \frac{C \Delta q_v(x_F|tr)}{q_v(x_F)}, \text{ at } x_F \rightarrow 1. \quad (5)$$

For small  $x_F$ ,  $D(x_F, M|s) \ll N_0(x_F, M|s)$ . Hence,

$$A_N(x_F, M|s) \rightarrow 0, \text{ at } x_F \rightarrow 0, \quad (6)$$

This implies that for  $x_F$  goes from zero to one,  $|A_N|$  should start from zero, begin to rise somewhere and reach  $C \Delta q_v(x_F|tr)/q_v(x_F)$  at large  $x_F$ . We also clearly see that where  $|A_N|$  begins to rise is determined by the interplay of the two contributions  $D(x_F, M|s)$  and  $N_0(x_F, M|s)$ . Whether it is dependent of energy is determined by the energy dependences of  $D(x_F, M|s)$  and  $N_0(x_F, M|s)$ .

Eq.(4) has been applied to calculate the  $A_N$ 's for different mesons in  $pp$  and  $\bar{p}p$  collisions at the E704 energy and the results are in agreement with the data. Now we use it to discuss the energy dependence of  $A_N(x_F, M|s)$ . We note that due to the scaling behavior of the inclusive cross section in hadron-hadron collisions in the central rapidity region[19], we expected that  $N_0(x_F|s)$  is independent of energy. The two parameters  $C$  and  $\kappa$  are two constants and are taken as independent of energy. There is an obvious energy dependence in  $D(x_F, M|s)$  which is caused by the energy dependence of  $x_0$ . This leads to an energy dependence of  $A_N(x_F, M|s)$  in the picture. We now analysis whether this energy dependence is in the right direction as that observed in experiments.

As can be seen from the expression of  $x_0(= m^2/s)$ , if  $\sqrt{s}$  is higher,  $x_0$  is smaller. Since in the small  $x$  region  $\bar{q}_s(x)$  increases rapidly with decreasing  $x$ , this leads to a rapid increase

of  $\bar{q}_s(x_s)$  (where  $x_s \approx x_0/x_F$ ) with increasing  $\sqrt{s}$ . This means that, at higher  $\sqrt{s}$ , the number of  $\bar{q}_s$  at  $x_s$  is larger so that the probability for the  $q_v$  to meet a suitable  $\bar{q}_s$  to form the meson  $M$  is larger. A direct consequence is that  $D(x_F, M|s)$  becomes more important compared to  $N_0(x_F, M|s)$ . Hence  $|A_N|$  should start to become non-zero earlier when  $x_F$  increases from zero to one at higher  $\sqrt{s}$ . This qualitative expectation is in agreement with the experimental observation. Now, we use completely the same inputs as that used in [7] for  $p_{inc} = 200$  GeV/c but change the energy  $\sqrt{s}$  to the E925 energy, i.e. take  $\sqrt{s} = 6.56$  GeV to calculate  $A_N$  from Eq.(4). The results obtained are given in Fig.1. We see that the qualitative feature of the data can indeed be reproduced [20]. The quality of the fit to the data depends on the choice of  $\Delta q_v(x|tr)$ . Since it is still unknown yet, in [7], a simple ansatz  $\Delta q_v(x|tr) \propto q_v(x)$  was used[21], where the proportional constants were determined by using the SU(6) wavefunctions, i.e.,  $\Delta u_v(x|tr) = (2/3)u_v(x)$  and  $\Delta d_v(x|tr) = (-1/3)d_v(x)$ . To see whether the quality of the fit at the E925 energy and that at E704 energy can be improved simultaneously by making a more suitable choice of  $\Delta q_v(x|tr)$ , we take a more sophisticated form of  $\Delta q_v(x|tr)$  to make a better fit of the E704 data and then apply to the E925 energy. More precisely, instead of  $\Delta q_v(x|tr) \propto q_v(x)$ , we introduce an extra  $x$ -dependent factor  $f(x)$ , i.e. take[22]  $\Delta q_v(x|tr) \propto f_v(x)q_v(x)$ , where  $f_u(x) = 1.2x$ ,  $f_d(x) = 2.6x^{1.5}$  [more explicitly, we take  $\Delta u_v(x|tr) = 0.8xu_v(x)$  and  $\Delta d_v(x|tr) = -0.87x^{1.5}d_v(x)$ ]. The results are shown in Fig.2. We see that the quality of the fit at both energies can indeed be improved simultaneously.

We see that without any further input or adjusting any parameter, we can reproduce the energy dependence observed by E925 compared with the E704 results. It reflects the energy dependence of the interplay of the direct-formation part and the rest. Encouraged by this result, we apply the picture to make predictions for experiments at even higher energies, much higher than the E704 energy, such as those at RHIC. To do this, we recall that the energy dependence of  $A_N$  in the picture[7] comes from that of  $D(x_F, M|s)$ . The latter depends on  $\sqrt{s}$  because, when  $\sqrt{s}$  increases,  $x_0$  is smaller so that  $\bar{q}_s(x_s)$  is larger and there is more  $\bar{q}_s$ 's suitable to combine with the  $q_v$  to form the meson  $M$ . Since the number of  $q_v$ 's in a proton is very much limited, it can easily be imagined that, at very high  $\sqrt{s}$ ,  $\bar{q}_s(x_s)$  is very large so that there is always enough suitable  $q_s$  to combine with  $q_v$  to form  $M$ . In this limiting case,  $D(x_F, M|s)$  should be independent of  $\bar{q}_s(x_s)$  in the sense that if we further increase  $\sqrt{s}$  we will gain nothing more. This implies that  $D(x_F, M|s)$  should be

merely dependent on  $q_v(x_F)$  but independent of  $\bar{q}_s(x_s)$ . More precisely, at very high  $\sqrt{s}$ , we should have[23],

$$D(x_F, M|s \rightarrow \infty) = \gamma_M q_v(x_F). \quad (7)$$

where  $\gamma_M$  is a constant. Hence, we have,

$$A_N(x_F, s \rightarrow \infty) = \frac{C\gamma_M \Delta q_v(x_F|tr)}{N_0(x_F|M) + \gamma_M q_v(x_F)}, \quad (8)$$

The constant  $\gamma_M$  has a clear physical meaning: it is the probability for  $q_v$  to hadronize into the meson  $M$  in the quark-fusion process  $q_v + \bar{q}_s \rightarrow M$ . If we, for simplicity, take into account only vector and scalar meson production with relative weights 3 to 1 due to spin counting, and take into account the suppression factor  $\lambda$  ( $=0.3$ ) for strange production, we have that,

$$\gamma_\pi = 1/[4(2 + \lambda)]. \quad (9)$$

We use this to do the calculations and obtain the results as shown by the dashed lines in Fig.2. We see that in this case  $|A_N|$ 's start to rise even slightly earlier than those at the E704 energy. This can be checked by future experiments.

In summary, we calculated the energy dependence of single spin asymmetries in inclusive pion production processes using the picture proposed in [7]. We showed that the energy dependence observed by BNL E925 Collaboration compared to the Fermilab E704 results is a manifestation of the energy dependence of the interplay of the “direct-formation” part to the produced meson compared with the rest. We made predictions for experiments at event high energies.

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- [18] In practice, since there is no data directly on  $N(x_F, s)$ ,  $\kappa$  was obtained by fitting one data point at large  $x_F$  of the available data on the invariant cross section  $Ed^3\sigma/dp^3$  at given  $p_\perp$  because approximately  $Ed^3\sigma/dp^3 \propto x_F N(x_F, s)$ . More precisely, the number densities  $N(x_F, s)$ ,  $N_0(x_F, s)$  and  $D(x_F, s)$  were all multiplied with  $x_F$  and a constant which relates the differential cross section to the number density to become their counterparts in the invariant cross section  $Ed^3\sigma/dp^3$ . For  $\pi$  at  $p_\perp = 0.65$  GeV/c, the corresponding result for  $x_F N_0$  is  $1.0e^{-2x_F^2 - 20x_F^3}$  mb/GeV<sup>2</sup> and that for  $\kappa_\pi$  is  $2 \times 10^{-4}$  mb/GeV<sup>2</sup>. See [7, 8, 16] for details.
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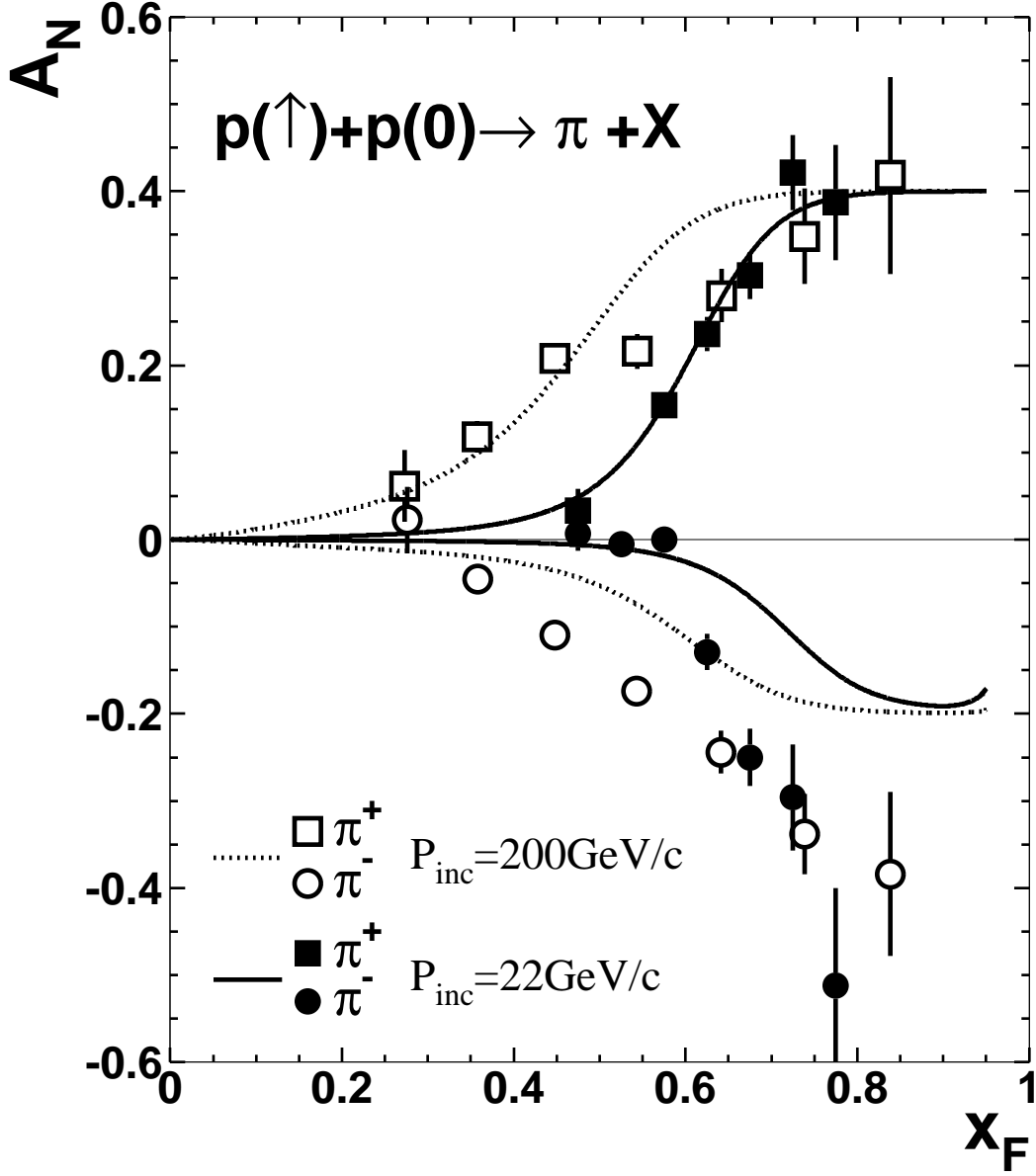


FIG. 1: Single-spin asymmetry  $A_N$  as a function of  $x_F$  for  $p(\uparrow)+p(0) \rightarrow \pi^\pm + X$  at  $p_{inc}=200\text{GeV}/c$  compared with those at  $p_{inc}=22\text{GeV}/c$ . The data are taken from [3] and [17] respectively. The theoretical curves are the calculated results from Eq.(4) using the simple ansatz for  $\Delta q_v(x|tr) \propto q_v(x)$  adopted in [7].

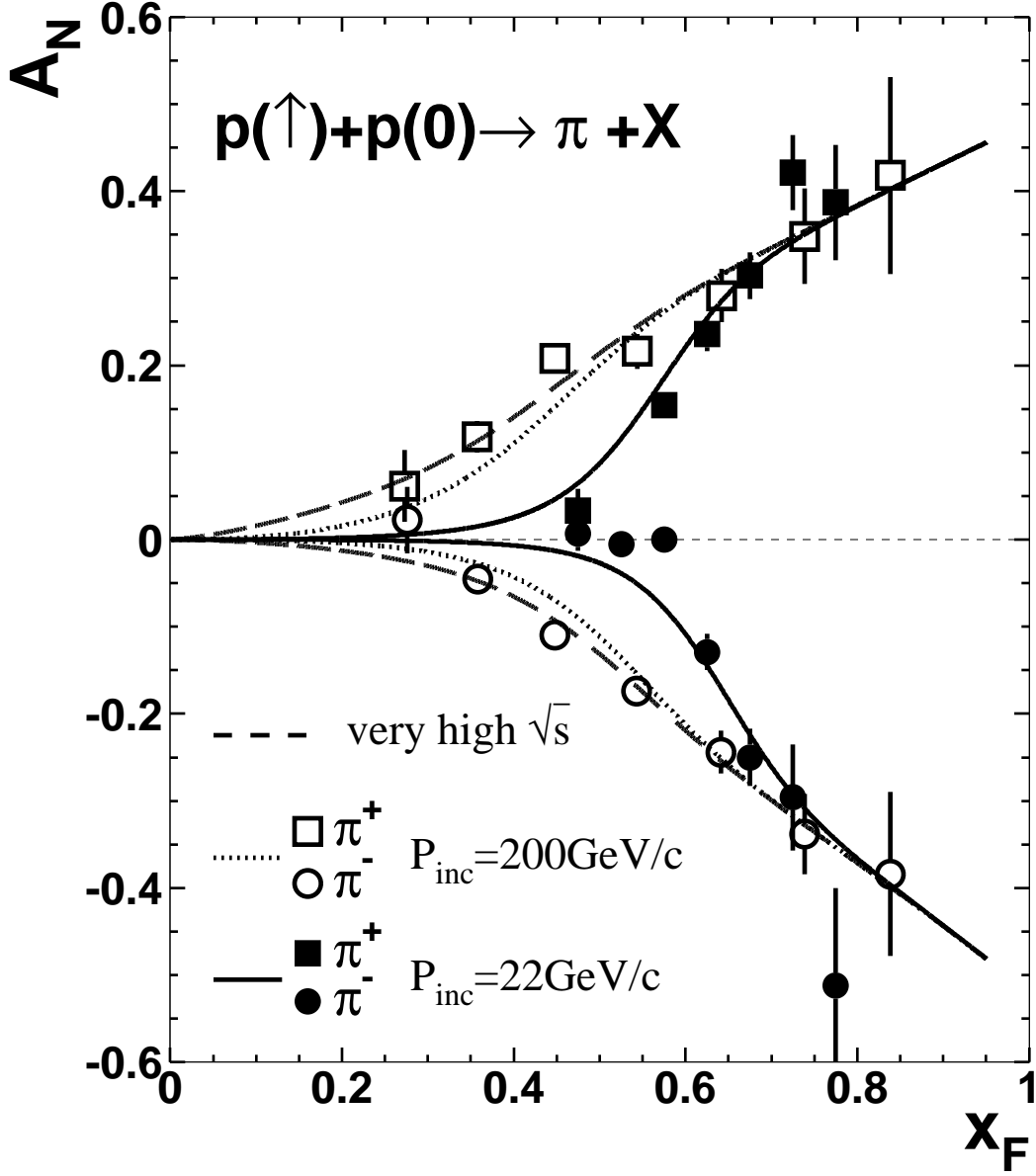


FIG. 2: Single-spin asymmetry  $A_N$  as a function of  $x_F$  for  $p(\uparrow) + p(0) \rightarrow \pi^\pm + X$  at  $p_{inc} = 200 \text{ GeV}/c$  (the two curves in the middle, dotted) compared with those at  $p_{inc} = 22 \text{ GeV}/c$  (the two innermost curves, solid) and those at very high energy (the two outermost curves, dashed). The data are taken from [3] and [17] respectively. The curves are the calculated results from Eq.(4) using a modified parametrization of  $\Delta q_v(x|tr)$ .